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Abstract

The large-scale networks of suppliers and buyers in industrial districts have rarely if ever been studied as social networks due to analytical complexity and rarity of datasets. We quantitatively analyzed such a complex system to identify its mechanisms of integration. Tests of the small-world model failed because of a power-law degree distribution, shorter-than-random average distances, and lack of local clustering. The scale-free network model was also rejected because primarily hubs organized the network not preferences of suppliers. A directed acyclic graph (DAG) model explained the structural properties. Finally, in lieu of small-world or scale-free models, we offer statistical evidence that the DAG should be a general property for the complex production networks, as modeled by Harrison White.

Key words: Complex systems; directed acyclic graph; flexible specialization; industrial districts; scale-free networks; small-world; structural embedding

INTRODUCTION: LARGE-SCALE INDUSTRIAL DISTRICTS AS COMPLEX NETWORKS

Industrial districts, while rarely studied as social networks, have been a contentious topic and one of increasing interest to scholars. For over the last 30 years, in hundreds of literatures, there have been heated debates among theorists and fieldworkers, concerning the structure and dynamics of localized industrial clusters, initially from Europe and later from other countries in all continents. One of the driving forces behind the rising interest has been "flexible specialization" theory proposed by Piore and Sabel (1984), which contends that in the relentlessly moving markets after the 1970s, nimble and flexible manufacturing systems in industrial districts that operate on the foundations of the division of labor among technologically specialized small- and medium-sized enterprises (SME) in regional networks[1] have a competitive advantage over the mass production system traditionally carried out by vertically integrated, atomized, large firms (Goodman and Bamford 1989; Lazerson 1995; Locke 1995; Pyke, Becattini and Sengenberger 1990; Sabel and Zeitlin 1997).[2]

While many researchers have endeavored to articulate mechanisms of "flexible specialization" in various industrial districts and time periods, those studies had limitations. First, existing empirical investigations mainly studied smaller-scale industrial districts where either a single or relatively few industries were involved, leaving large-scale industrial districts largely outside the research area.[3] These are systems whose complexity goes well beyond "flexible specialization" to produce numerous parts, components, and modules for a variety of end products. The local SME suppliers should differentiate in the division of labor under the leading roles of organizing prime buyers, in the tiers of subcontracting networks that eventually lead to the top original equipment manufacturers (OEM) for the assembling work as hubs.[4]

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[1] We use the term SME rather loosely throughout our discussion, as the concept can carry various definitions. For instance, the definition of the Official Journal of the European Union and the counterpart of the official public statistical data by the Japanese government differ in term of the number of employees. The term therefore generally encompasses micro-, small- and medium-sized enterprises.

[2] "Flexible specialization" theory generally includes social aspects of division of labor among stakeholders as an institutionalized role structure embedded in each of the regional economies. The conception encompasses not only the division of labor among the SMEs engaged in the manufacturing per se but also other actors filling important roles, including administrative and political supporting institutions, banks and other financial institutions, family networks of proprietors, public training institutions and private educational initiatives, and industrial associations. However, as our network analytical study focuses on the integration mechanisms of the regional production system, employing interfirm trade network data, we use the division of labor in a narrower sense as to refer to the supplier-prime buyer relationships.

[3] There are a few exceptions (Lane 2002), and ongoing projects in process include "Distretti industriali come sistemi complessi" with principal researcher Margherita Russo.

[4] We do not differentiate such terms as "OEMs", "leading manufactures," and "top firms in the hierarchy of supplier-prime buyer relations" throughout our discussion. Literally speaking, an OEM is original equipment manufacturer, or a firm that produces end products, which are purchased by the consumers possibly under different brand names. In other words, these are the actual assemblers. To give an example, in year 2004, Sanyo is known as a major OEM for many digital cameras sold under different brand names worldwide. Practically, however, it is almost impossible to tell the real OEMs behind the top brand names, as each brand consists of so many different products manufactured by different OEMs. In the case of the complex supplier-buyer networks, any supplier that

Second, studies of regional interfirm networks relied mainly on qualitative research techniques in order to depict the structural integration mechanisms, and these findings have evoked considerable debate. While those studies qualitatively provided rich details about the competitive advantage of the SMEs' social capital, they were not able to communicate systematically the structural mechanisms of industrial districts.

Finally, given the conceptual framework as the division of labor among technologically specialized SMEs through regional ties in rather cohesive networks, 'flexible specialization' implicitly put focus on local structure, or the smaller parts of whole networks. It was conceptually beyond the scope to discuss global, or overall, integration mechanisms of the complex networks.

In effect, the complex webs in large industrial districts are understudied, as both the overwhelming technical complexities and the rarity of such network datasets put technical limitations on researchers wanting to carry out quantitative analysis of the interfirm networks. There is today a paucity of alternative theories available with which to explore and explain the network properties.[5]

COMPLEX INTERACTIVE NETWORK MODELS

Social network analysis has a long history (Freeman 2004; Mullins and Mullins 1973), but the study of complex interactive networks has gained attention in many academic disciplines especially since the late 1990s. Network models in sociology became increasingly mathematized in the small group studies of the 1950s that focused on cohesive cliques in order to identity sources of emergent norms in society (Homans 1950). Later attempts by Blau (1977), as a further example, were mathematized by Fararo (1981) using biased-network theory to explain social structure in complex society on the foundations of Simmel's ideas of intersecting social circles in urban life (1955). As a more recent example in the area of knowledge management, Powell and his colleagues (2005) studied the evolution of large-scale structural cohesion and integrative diversity in collaboration networks among firms in the life sciences, as complex interactive networks. Furthermore, in anthropology, White, Schnegg and Brudner (1999) conducted a large-scale analysis of marriage, sponsorship and elite networks in Mexico, finding a two-level "invisible state" that bound together districts in a large geographic region, reinforcing a distinctive cultural heritage.

Among the alternative models for complex interactive networks, studies of small-world (Watts 1999a) and scale-free network (Barabási 2002) represent breakthrough achievements that have drawn much recent attention. Watts (1999a) revitalized forty year-old research questions as to explain how a large number of highly-clustered nodes in a large-scale but sparsely-connected network (LSN) can be connected in relatively few steps, as for example, by only "six-degrees of separation" (Milgram 1967; White 1970). This offered a powerful theoretical model to account for the ubiquity of large but small-world networks, such as those that

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assembles products for a buyer sitting at the top of the subcontracting hierarchies can be an OEM when the top buyer puts its own brand name on the supplied end-products. Therefore, we rather call these buyer/suppliers OEMs in the aggregate. Any large prime buyer as an organizing hub can be an OEM for another prime buyer. Most large prime buyers are probably OEMs for other large prime buyers that have reputable brand equity.

[5] For a summery theoretical discussion regarding the historical changes in the manufacturing technologies related to industrial clusters, see for example Trigilia (2002:198-218).

Watts and Strogatz (1998) found in a variety of social, biological, and technological phenomena, ranging from collaborations among Hollywood actors to power grid connections or neural networks of a microbe. Many other applications of the small-world model have been made recently, including Newman's study of scientific collaboration networks (2001), and organizational studies in the areas of corporate interlocks and governance structure (Davis, Yoo and Baker 2003; Kogut and Walker 2001; Robins and Alexander 2004), interfirm alliance formation and joint ventures (Baum, Shipilov and Rowley 2003), and emergence of industries (Uzzi and Spiro 2005).

Recently Barabási (2002) critiqued Watts' small-world models for their failure to take into account some of the hierarchical properties of network hubs. Instead, he proposed scale-free network models as congruent to some of the more ubiquitous forms of centralization in networks. They are designed for networks in which the relative frequencies of hubs are inversely proportional to some exponential power α of each node's degree, freq(node i of deg(i)) ~ $1/\text{deg}(i)^{\alpha}$. Barabási's model proposes a generative process for the formation of links between an equiprobable source node that connects to target nodes with variable probabilities. The main model consists of a simulation with a generating probability function $P(i) \sim \text{deg}(i)$ for the probability of connection to a target node i that is proportional to its degree, which is taken to represent its attractiveness. He shows that this very simple model of preferential attraction would account for the range of observed power-law degree distributions in empirical networks. The coefficient α for resultant empirical networks is expected to vary in this model between 1 and 3 as a function of the number of nodes and edges generated in the network, plus random noise, converging on $\alpha = 3$ for the case where a sparse network grows toward infinite size.

Debates about small-world and scale-free models are illustrative of how the study of complex networks, while rapidly expanding, is fraught with open questions, especially about the relation between network growth and decline, structure and change, and the underlying processes that generate structure and the dynamics driving structural change. The Barabási model has been severely criticized, for example, because it is but one of many ways that power-law degree distributions might be generated. Further, the model assumes that formation of edges from each node takes into account the current degree of every other node, a very implausible assumption. Network realism would require restrictions on edge formation, such as dependence on local context in terms of indigenous culture and knowledge as well as the external environment and institutional settings. Past and current examples abound that are relevant to such issues in the study of complex interactive networks. But, for large-scale industrial districts, only a very few network studies have yet been made.

The objective of the present research, therefore, is to analyze structural properties of embedded supplier networks in a large-scale industrial district, employing recent innovations both in theory and quantitative analytical techniques in the area of complex interactive networks. While small-world and scale-free models are often seen as the major contending models for ubiquitous types of networks, in reality they offer a very limited set of alternatives. In the present paper, we take up the applicability of these two models when tested against our industrial district data before turning to other possibilities. We find, empirically, that neither model provides a good fit to the data nor a better understanding of industrial districts. Network realism requires that we push further than these alternatives if we wish to understand sociological and economic processes involved in the formation of industrial networks. We then turn to alternative models and analyses that are more sensitive to the hierarchical structure of production networks. Specific hypotheses are tested for empirical fit to models of acyclic hierarchy, structural cohesion, triads census frequency, and network robustness. Finally, we conclude with what we have learned from modeling the general network structure of buyer-supplier relations in large industrial districts, and some of its dynamics, as an empirical study of the producer market theorized by Harrison White, exploring discussions what might be done further, had we better data on the longitudinal development of an industrial district network.

RESEARCH DESIGN, DATA, AND ANALYSIS

The research questions of this industrial district study are oriented toward identifying possible structural patterns and dynamics that might be general or at least not uncommon across the complex interactive webs of supplier-prime buyer relationships in large-scale industrial districts. Because "flexible specialization" enjoys the greatest currency in contemporary theoretical discussions of industrial clusters, we also ask the following questions throughout: Are there any distinctive structural patterns peculiar to the supplier-prime buyer networks? Can we generalize to other industrial districts? *Pajek* (Batagelj and Mrvar 2005) was used both to calculate network analytical measures and draw graphs.[6]

SMALL-WORLD ANALYSIS

Small-World Mechanisms

Are large-scale industrial districts small-world in their organization? That is, are the supplier-prime buyer networks organized in ways that not only connect large numbers of firms but do so for clusters of firms that are more closely connected, while retaining relatively short distances between pairs of firms in the network overall? A small-world is a large, sparse, clustered network that has, surprisingly, relatively short average distances between nodes (Kochen 1989). This occurs when actors in a sparse and clustered network get connected by a small number of intermediaries, or in relatively short steps, so as to span the nodes in other clusters. Watts (1999b:496-97) offered an explanation how a small-world can emerge, as follows:

1. The network is numerically large in the sense that the world contains *n* nodes.
2. The network is sparse in the sense that each node is connected to an average of only <*k*> other nodes, which is on the order of thousands or hundreds of thousands of times smaller than the population *n*. While sparse, the network should have a large component of interconnected nodes.
3. The network is decentralized in that there is no single dominant node to which most others are directly connected. This implies a stronger condition than sparseness, as not only must the average degree <*k*> be much less than *n*, but the maximal degree over all nodes must also be much less than *n*.
4. The network is highly clustered, in that most social circles are strongly overlapping. That is, connected triads are expected to become fully connected triples, as exemplified in the case where many of our friends are also friends of each other.

[6] The software is available from http://vlado.fmf.uni-lj.si/pub/networks/pajek.

These preconditions of small-world may easily produce a network where the average path length between any two randomly selected nodes from the population is quite long. Watts (1999a) found that even when as little as one percent of the paths in the network are randomly rewired to create new ties, the network begins to exhibit small-world phenomena where a very large number of nodes are connected as a component but still with high local clustering and a relatively short average path length among the nodes. Even at low densities, once created, the small-world is resilient and robust to changes that disrupt some of its connections (Davis, Yoo and Baker 2003; Kogut and Walker 2001:321; Watts and Strogatz 1998). As many kinds of social and economic structures, however, place strong constraints on the generation and placement of edges, small-world properties cannot be taken for granted.

Ohta Industrial District, Data and Small-World Logic

The present research studied a web of supplier-prime buyer relationships among manufacturing firms linked to Ohta, which is one of 23 wards in Tokyo, as one of the two largest industrial districts in Japan. Over 7,000 SMEs were engaged in a variety of manufacturing processing activities, parts, components, and modules production, and assembling work to compose a complex web of regional interfirm linkages. A majority of the SMEs were the size of a typical family household, or even smaller, in terms of the number of employees. The industrial district has been well known as a so-called machine-tools industry where the SMEs functioned as suppliers for leading Japanese OEMs in other applied industries.

At the time of the survey, in 1994-95, among over 7,000 manufacturing firms in the industrial district, a majority of firms were specialized in their own specialized areas of manufacturing processing activities. In particular, many firms were engaged in various metal-cutting processes. At the same time, a minority were suppliers of parts and components in areas such as automobile production, aerospace technologies, computer-related products, electrical and electronic equipment and devices, general industrial and precision machinery, jigs and tools, and shipbuilding, among other areas. Roughly only 10%-20% of suppliers that specialized in certain areas of processing and parts and components production had product lines of their own brands.

To conduct the present network analysis, name-generating data from *Akusesu Data* (Ohta-ku Sangyo Shinko Kyokai 1997a, 1997b) were used. The dataset encompasses approximately 70% of all manufacturing establishments in operation in Ohta-ward during the years of 1994-95. The questionnaire employed in fact asked each of the roughly 7,000 SMEs located in Ohta-ward to list up to three names of their prime buyers. To be specific, among the 5,111 firms in Ohta from the dataset, 2,710 firms (53%) listed a total pool of 4,077 other firms as their prime buyers. Another 2,401 firms (47%) listed no prime buyers. Of the 5,111 SMEs that responded, 501 firms (9.8%) listed only one prime buyer; 530 firms (10.4%) only two; and 1,679 firms (32.9%) listed three names as their prime buyers. Of the 4,077 listed prime buyers, 841 were supplier-prime buyers located in Ohta, which were named by peer SME suppliers in Ohta, and 3,236 were prime buyers outside Ohta. The total number of firms in the dataset and included in the network was 8,347.

The large-scale regional interfirm networks in Ohta were hypothesized to fit the preconditions of the small-world, due to the following reasons. First, the number of firms in the large-scale regional supplier network was large with 8,347 firms in the dataset. Second, it is likely to have a decentralized network structure because, in the large regional web of supplier-prime buyer network where a variety of manufacturing industries were embedded, it would be unusual for a single firm to dominate the whole web. Third, assuming that the SMEs working in lead OEMs' local supplier groups, and that they had relationships with one another, there should be many densely connected local clusters within the complex web. Fourth, the complex web should be regionally connected to form a large component so as to link a variety of manufacturing processes and flows of goods and services, as a sort of industry value-chain. Finally, thus there should be enough shortcuts in the connected component(s) so that average distances between nodes are minimal, comparable to a random graph. Thus, the first hypothesis:

Hypothesis 1: Complex webs of regional supplier networks in large-scale industrial districts are small-world.

Small-World Numbers and Analysis

Applying the small-world preconditions to the case of interfirm trade relationships in a large-scale industrial district, the following conditions were tested to see whether the network actually meets the criteria:

Condition 1. The regionally embedded web of supplier networks should be large in the number of firms contained but there should be a large component where firms are connected. Further, the average shortest path distance between firms should be low, or close to the minimum, as is true for random networks.

Condition 2. The network should be sparsely connected in the sense that each node is connected only to a small number of other nodes, relative to the number of the firms contained in the regional interfirm network.

Condition 3. The complex web should be decentralized in that there is no single dominant firm to which most other nodes are directly connected or adjacent. The highest degree-centrality of a node should be small relative to the size of the network.

Condition 4. The complex regional web should be highly locally clustered under the lead OEMs. Specifically, the suppliers for each OEM should have extensive interactions or interfirm trade relationships among themselves.

Condition 5. The above four conditions should meet simultaneously so as to create small-world.

In considering paths and path-length in terms of the small-world model, the direction of supplier-buyer links may be ignored, and each connected pair of firms is considered to constitute a symmetric link. As any exchange of goods through directional ties is most likely to involve at least some communication, or two-way information exchange between the two partners (Freeman 1979; Hanneman 2001), the weakly-connected components should bear social implications of the supplier-prime buyer relationships.

Regarding Condition 1—Presence of a large component with its relatively short average path length: Component analysis was conducted to find the largest component in the dataset, and its average shortest path length in geodesic was measured and compared to the average expected for random networks. A *component* is a maximal connected sub-graph in which all the nodes are connected to one another through one or more paths.

To test Condition 2—Formation of a sparsely connected network: The data structure created preconditions for a sparse network, as it includes information on only up to three names of prime buyers listed by each of 5,111 SMEs. As the small-world formula takes into account the total number of firms in the network as *n*, and the average <*k*> of numbers of firms listed by each firm, the limitations of the data structure were factored into the formula.[7]

The network met Conditions 1 and 2, as the largest component was identified as having 4,500 firms, excluding 3,847 firms disconnected from the component. The average path length was 8.86, which is unusually short for such a sparse network, and is even shorter than expected from a comparable, totally rewired, random graph. Greater shortest path efficiency than a random network is <u>not</u> a small-world characteristic.

With regard to Condition 3—No single firm dominance of the network: Degree-centrality of prime buyers was employed to test the criterion. Node centrality has been a key analytical concept in social network analysis (Freeman 2004). The in-degree centrality score of a firm was determined by the aggregate number of times the 5,111 SME suppliers listed a given firm as one of their three prime buyers. Toshiba was the most popular prime buyer, which means that 112 suppliers in Ohta listed Toshiba. Arguably, one could cite Toshiba with an in-degree of 112 suppliers, but NEC has 53, Hitachi 45, and so forth, so that there are many hubs along a gradient, not just one.

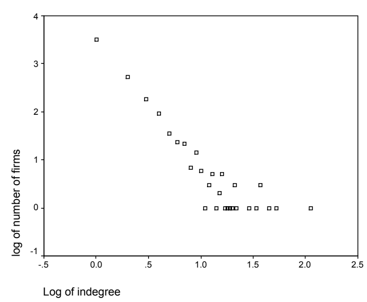

Fig. 1.—A power law in Ohta

Note.—Data was created from *Ohta-ku Akusesu Deta 1 & 2* (Ohta-ku Sangyo Shinko Kyokai 1997a; 1997b).

According to Watt's criteria, however, there is no single dominant node to which most other nodes are directly connected. To be specific, average degree <k=1.58> of the network is several thousand times less than n_{all} =8,347, and the maximal degree k_{max} =112 for nodes in the network is nearly a hundred times less than n_{all}. Further, the various indices of graph centralization for the largest component proposed by Freeman (1979)—degree, betweenness, and closeness centralization—are all extremely low. All these indices, however, regard a single hub with spokes as the model of a centralized network, rather than a scale-free model of hub centralities. The network fulfilled Condition 3 in that there was no single firm constituted a hub that dominated the large-scale regional web, but only marginally so.

Contrary to the small-world and decentralized model, however, there was a "pecking order" for frequency of ties to powerful OEMs, consistent with the kind of power law that Barabási (2002) uses to characterize

[7] We also show in Appendix C that this sparsity remains in survey questions concerning all buyers from other surveys, not just the top three prime buyers, and in estimates of the full "prime buyer" links beyond the three listed.

preferential attachments to hubs proportional to degree in a scale-free network. The distribution of in-degree for all firms is plotted in Figure 1 on a double-logarithmic scale. The slope ($\alpha \sim 2.3$) of the straight line that approximates the distribution ($y = 1539.4x^{-2.2862}$ and $R^2 = 0.8537$) is within the range of values ($\alpha \sim 1.8$ to 2.5) for scale-free preferential attachment networks of size 4-8,000 (White and Johansen 2005:17).[8]

As to test Condition 4—A high local network-clustering coefficient: While improved calculations for a local cluster coefficient were given by Newman, Watts, and Strogatz (2001) and Newman (2003), as the average probability that two neighbors of a given vertex are also neighbors of one another, the present analysis followed the cluster calculation used in the previous research studies of corporate interlocks including by Davis, Yoo and Baker (2003:fn.3) and by Kogut and Walker (2001:324), which is the average of the densities of all ego networks deleting adjacent ties between the ego and the alters, and can also be characterized as the number of fully connected triads over connected triads, ignoring directions of the edges.

To test Condition 5, Watts and Strogatz (1998) provided a general formula to detect whether clustering and low average distance in large sparse networks (LSN) meet simultaneously so as to create small-world: Summary small-world numbers were calculated for the largest component based on the formula used by Davis, Yoo and Baker (2003:313) and by Kogut and Waker (2001:324). The large-scale component consisted of n =4,500 nodes where each of nodes had an average of $<k=2.3849>$ direct ties with other nodes in the network. L_{actual} is the average shortest path distance in geodesic between two nodes counted across all combinations of nodes in the largest component. L_{random} is the average shortest path length between two nodes calculated across all combinations of nodes in the component, given that the ties between the nodes are randomly assigned, which can be approximated by $\ln(n) / \ln(<k>)$, or 9.7695. C_{actual} equals 0.0000517, or the actual clustering coefficient calculated as the average density of ego networks of all the nodes in the component, when all direct ties between the ego and alters, or neighbors of the ego with path length of 1, are excluded from the computation. C_{random} equals the average of local clustering in the randomized network, which can be approximated by $<k> / n$. Finally, the small-world measure is the product of two ratios: The actual to randomized average clustering (which should be large) times randomized to actual average geodesic path length (which should be smaller than 1 but not too small). If the ratios are respectively greater than 1 and less than 1, and if their product is substantially greater than 1 (clustering weighting more than distance), the component is small-world. In sum, for a network to be small-world, its average geodesic distance should be only slightly greater than $\ln(n) / \ln(<k>)$, when an approximate average distance for a random network has no degree bias (Durrett 2006:7), but its actual clustering coefficient substantially greater than $<k> / n$.

Table 1 compares the Ohta fit to the small-world model. As a cautionary note, as the clustering coefficients for unipartite networks tend to be very small relative to its counterpart for bipartite networks (Newman, Strogatz and Watts 2001), simple comparison of coefficients for the bipartite networks with the unipartite Ohta network can be misleading. The small-world measure for the Ohta largest component of 0.107, as shown in Table 1, in failing the small-world test, is much lower than that of the simplest of biological organisms at 4.75, and a hundred times less than that of the power-grid technological network at 10.61.

[8] We follow here the fitting procedure of Goldstein, Morris and Yen (2004) to fit to the lowest bins that contain the bulk of the sample observations in a power-law distribution, instead of the simple linear fit.

TABLE 1

Small-World Numbers in Comparison

Network	N (number of nodes)	N (number of nodes in component)	<K> (average degree)	Average Path Length		Clustering Coefficient		Small-World Measure
				L (actual)	L (random)	C (actual)	C (random)	Actual-to-Random Ratio for Clustering/Length
bipartite								
Hollywood film actors network (bipartite)	212,250	u.k.	28.78	3.65	2.99	0.79	0.001	2396.9
American corporate director interlock (1999)	5311	4538	16.00	4.33	3.06	0.87	0.003	183.03
German firms, connected	538	291	u.k.	5.46	3.01	0.84	0.022	22.46
German owners, connected	429	u.k.	u.k.	6.09	5.16	0.83	0.008	100.48
unipartite								
C. Elegans network	u.k.	282	14	2.65	2.25	0.28	0.050	4.75
Power grid network	u.k.	4941	2.7	18.70	12.40	0.08	0.005	10.61
Ohta interfirm networks	8347	4500	2.3849 Sym.	8.86.	9.77	0.0000517	0.00053	0.107

Note.—Data adopted from the following sources, except Ohta data: Watts and Strogatz (1998), Kogut and Walker (2001), and Davis, Yoo and Baker (2003). Numbers for Ohta interfirm networks calculated from *Ohta-ku Akusesu Deta 1 & 2* (Ohta-ku Sangyo Shinko Kyokai 1997a; 1997b). Simulations with random variables for the two Ohta networks gave average path lengths of 11.1 and 6.29, in the range of the approximation formula $\log(n)/\log(<k>)$.

While the small-world hypothesis was rejected because of lack of clustering, there were some interesting findings. First, as the small-world measure for Ohta suggests, the actual ratio of the average geodesic distance to the average local clustering (C_{actual} / L_{actual}), or 0.000005835, is one-tenth of the randomized ratio (C_{random} / L_{random}), or 0.000054247. Second, as the network had a very small clustering coefficient, C_{actual} = 0.0000517, one-tenth that of the clustering coefficient of the randomized network, C_{random} = 0.000530, the absolute level of the actual clustering was extremely small compared with the ones of other small-world networks. Finally, the component had a short actual average geodesic distance of 8.86, even shorter than the estimated average geodesic distance, L_{random} = 9.77, for the same network with randomly rewired ties. However, partly because of the low average degree of nodes in this network, the absolute level of the average distance of 8.86 was relatively long, compared with the figures of the other small-world networks.

Why did the small-world model fail in the Ohta case and what are the implications of these failures? The shorter path length compared to the random expected figure needs to be explained by some other model, such as central hubs. Also requiring explanation is the relatively low level of actual average local clustering.[9] This

[9] The triads census that we will use later to evaluate network structure provides an expected probability model for clustering in fully connected triads which use different criteria for statistical independence. In Table 2, for example, the expected value of the 030 T and 030 C triads, which are fully connected, is 2.25 for the Ohta large component, compared to 13 actual. This would suggest that there is some residual clustering in the Ohta data, but 13 triads out of 25,413 connected triples (the observed rate of 0.0000517 in Table 1), is a very low rate.

indicates that ties among the adjacent neighbors for a lead OEM, which we call Tier-1 suppliers, were largely missing. In other words, Tier-1 suppliers were not trading with other Tier-1 suppliers of the same OEM or other firm but then neither were the Tier-2 suppliers located two links down from the lead OEMs. The lack of the interfirm links among the Tier-1 suppliers and lower tiers is consistent with the concept of "competitive cooperation" in the division of labor among suppliers in industrial districts, as proposed by "flexible specialization" theory (Sabel and Zeitlin 1997).

The lack of clustering owing to the "competitive cooperation" is illustrated by Figure 2, for NEC's hierarchically organized "double hub-and-spoke" network structure in Ohta, extending to include 124 supplier firms at Tiers-1, -2 and -3. Ranked second in terms of in-degree from its 53 Tier-1 suppliers, as a leading OEM in Ohta, NEC used local-core Tier-1 firms as assemblers or organizers of parts and components production. While the Tier-1 suppliers did not supply one another, each lower tier also used independent subordinates as Tier-2 or lower in order to organize the division of labor. Similarly, to a certain degree, the supplier group networks of other large OEMs operate on the foundations of this kind of division of labor among specialized SME suppliers in their areas of technological competence, engineering knowledge, and workers' skills.

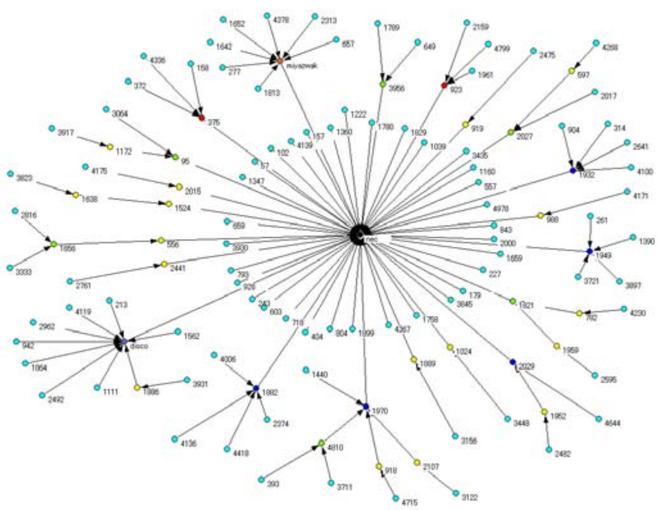

Fig. 2.—NEC's supplier network in Ohta: a local structure.

Note.—Graph produced from data in *Ohta-ku Akusesu Deta 1 & 2* (Ohta-ku Sangyo Shinko Kyokai 1997a; 1997b). Colors by in-degree.

Condition 3 specified by Watts (1999b) as to whether one node takes up the lion's share of all the links in the network, which was also followed by Davis, Yoo, and Baker (2003:312) and Kogut and Walker (2001:324), is clearly inadequate.[10] In fact, Barabási (2002) suggested other criteria for centralized networks, namely, whether the network was scale-free in terms of having a degree distribution that follows a power law. If we add Barabási's criterion to that of Watts' decentralization criterion, then the Ohta supplier network fails

[10] To reiterate for clarity, Watts' conception that a small-world should be decentralized is not adequately specified by the idea of "no single dominant firm," which he evaluated by the highest degree centrality among the nodes (or average degree) relative to the *n* nodes in the network, much like the notion of degree centrality as defined by Freeman (1979).

three of the four criteria for a small-world network, having only passed the sparseness condition to form a large component.

Although the distribution of the node links shows a power law as suggested by Barabási, the model does not fit the logic of the supplier-prime buyer networks, as the OEMs as hubs play the organizing roles in the complex webs, rather than preferential attachments from the suppliers to hubs. Knowing that there is power-law degree centralization in the network also suggests that models of hubs and hierarchy should be investigated further.

THE GLOBAL STRUCTURE OF "FLEXIBLE SPECIALIZATION" IN LARGE INDUSTRIAL DISTRICTS: AN ALTERNATIVE EXPLANATION

As the Ohta's large-scale network neither fit the small-world nor the scale-free criteria, the next critical questions are as follows: How did the complex web of regional supplier networks generate not only a giant component of 4,500 firms, but one in which the average shortest distance is even more efficient or shorter than that of a random network? What generates its power-law degree hierarchy? What is its structure, if not small-world or scale-free? How likely is the occurrence of this structure in other industrial districts?

Industrial District Structure as Oriented Graphs and Acyclic Depth Partitions

In the small-world model, direction of ties was ignored when making the tests of clustering, shortest paths, and lack of centralization. Directedness is not considered important in the small-world model. For the scale-free model, although we used directionality to test for an in-degree power law, attachment is not the relevant generating process. In either case, strict asymmetry was neither ruled in nor ruled out by the way data were collected. In point of fact, however, not a single pair of firms named one another as prime buyers in the Ohta data. The network of firms thus constitute not just a digraph, or graph in which the direction(s) of links are indicated, but an *oriented digraph* (Harary 1969: 10), or a digraph with no symmetric pair of directed links. We use *oriented network* to refer to this property of the subcontracting supplier-prime buyer network. This property allows consideration of alternative hypotheses that apply to the class of oriented networks

Why should one-way directedness of supplier-prime buyer links be taken as one of the fundamental characteristics of Ohta's complex web? The supplier-prime buyer relations in an interfirm supply-chain must be oriented along the flows of the linked processing activities in the hierarchies—initially from raw materials; then processed parts and components; next manufactured modules; and finally the assembled finished products by OEMs. This however moves in the direction opposite to the revenue streams that come from the sales of the goods and services, as the consumers at large are customers who inject money in the industry value-chain, as revenues shared successively by the subcontracting manufacturing firms through the chains of exchange in the hierarchy. The hierarchy is organized from the interaction of downstream consumers of the end products with OEMs to the upstream SME suppliers.

Some oriented networks have hierarchical properties, others do not. An *acyclic network* is an oriented network of a special kind that contains no directed cycles. If we start a path from any node in the network and follow the direction of arcs (directional lines), there is no return to the node of origin. When there is no cycle in the network, the acyclic relationships among the nodes in the network constitute, by proof following from definition, a *depth hierarchy* (Harary 1969:200). The hierarchical property follows only because nodes can be partially ordered by their emergent appearance in directed paths. Nodes in a graph that can be partially ordered in this way will have an adjacency matrix that can be ordered so as to be *upper triangular*, with 1s above the diagonal describing all the directed paths, and only zeros below the diagonal.[11]

In a large-scale regional cluster like the one in Ohta where thousands of SME suppliers provide their goods and manufacturing services to local hubs and powerful OEMs, it is very unlikely to find the directed cycles in the supplier-prime buyer network. The recent advancement in information and manufacturing technologies after the 1990s has made the OEMs leaner, more efficient and more powerful than ever (Helper, MacDuffie and Sabel 2001), taking advantage of the collaborative arrangements with their SME suppliers (Kristensen and Zeitlin 2005) as "mass customization" or "diversified quality production" (Streeck 1992). As a result, large OEMs can organize an efficient flexible production system for both volume and variety, which never used to be compatible in terms of the production costs.

The implications of the change upon the network integration mechanisms are as follows. First, it mitigates against exchange cycles. As the SMEs lack financial capital to invest in the very expensive high-tech equipments and advanced machining technologies, these SMEs as dedicated suppliers need to depend upon the controlling, powerful OEMs, belonging to the efficient but hierarchical production systems in order to get access to the advanced management science techniques and information technologies, such as just-in-time (JIT) inventory control, supply-chain management, total quality control, and concurrent engineering, among others.

Second, a role structure in the network on the basis of the complex value-chain largely precludes formation of directed cycles. Two kinds of division of labor are embedded in the complex network as the enmeshed role structures: A horizontal configuration among suppliers organized by prime buyers or hubs, as traditionally claimed by "flexible specialization" theory; and vertical as linked flows of manufacturing processes, marketing and sales, and distribution stages. A series of horizontal division of labor among suppliers organized by hubs are vertically linked or oriented as entangled chains of production flows towards the assembly work by OEMs,[12] and the subsequent marketing and sales and delivery stages follow. As the consumer stands outside the set of linked manufacturing processes, the emergence of a production hierarchy is simply an end-point of a process in which cycles of exchange are shrinking to a vanishing point where the marketing and distribution stages by reputable large firms begin with their brand equity, involving a redefinition of the categories and statuses of agents in exchange. Due to the fact that the network is both oriented and acyclic, an acyclic hierarchy should result, *by mathematical definition,* but substantively out of an emergent process.

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[11] The *adjacency matrix* of a network is a square matrix filled with 0s and 1s, with 1s only for ordered pairs of nodes that are adjacent, as where a supplier names a buyer. If there is a single pair of nodes with that are reciprocally adjacent the matrix cannot be upper triangular. Similarly if there is a directed cycle.

[12] This configuration is also a deepening process of manufacturing specialization by SME suppliers dedicated to narrow areas of their own processing activities in the tiers of subcontracting. The number of suppliers involved at each of the stages should decrease gradually as the process moves from the bottom to the top, owing to the coordination and network integration by the organizing prime buyers as hubs.

Hypothesis 2: Directed cycles are absent or extremely rare in the LSNs of complex supplier-prime buyer relations in large-scale industrial districts where OEMs are filling organizing and integrating roles as hubs.

Hypothesis 3: The dominant structure of regional supplier-prime buyer networks in large-scale industrial districts under the organizing roles of OEMs as hubs is DAG.

While the definition of an acyclic network is derived from the notion of directed cycle, there are relatively few social science studies that fully employed these concepts.[13] Hage and Harary (1996) applied acyclic digraphs and level assignments to sociopolitical systems in Oceania, and many more examples could be given, but there are few large-scale network analyses outside of kinship studies[14] that make use of acyclic networks and depth partitions as an element of social structure.

Assigning rank to partially ordered elements or nodes in an upper triangular adjacency matrix can be easily accomplished by construction of an acyclic depth partition that is based on two repeated steps. First, all nodes that do not have any in-degree are assigned to level d=1. Second, these nodes and their outgoing lines are removed. Steps one and two are then repeated to identify level d=d+1 until no nodes remain (Batagelj and Mrvar 2005). The algorithm reorganizes the network into hierarchical levels such that if a node at any level has any incoming lines, at least one must be directed from the next lower level, as the node it connects to would drop down one or more levels otherwise. The analysis of levels in an acyclic graph is thus an effective method by which hierarchical structures can be detected even in an extremely complex network.[15]

When directed cycles are absent or rare, acyclic depth partitioning is robust to capture or approximate a great deal of the large sparse network (LSN) structure where links tend to be oriented. For industrial districts, the important comparative finding that would make our hypotheses broadly applicable is an overwhelmingly directed orientation of links in supplier-prime buyer networks. In a large network, sparseness of links may be sufficient, by itself, to set up statistical expectations that there will be few or no directed cycles. A large sparse oriented graph with an average of fewer than two ties per node is likely to have very few directed cycles, as explained in Appendices B and C.

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[13] As representational structures, however, DAGs are much-used in computer science and mathematical concept lattices. Simplified DAGs are used to classify, draw organizational trees and represent abstract hierarchies.

[14] Studies of directed cycles in social organization often build on ideas from anthropological exchange theory and structuralism, Hage and Harary (1996) identified acyclic exchange cycles that bound together cultural networks among South Pacific islanders, and Bearman (1997) leveraged the concept of directed cycles for network investigations in proposing a structural theory of generalized exchange. For a literature review, see Douglas White (2004) and White and Johansen (2005). Few network researchers, however, have followed their lead. Among the exceptions, Robins, Pattison, and Woolcock (2005) attempted to simulate an emergence of small-world, applying the analytical concept of cycles. Studies in the relatively new field of network economics (Plott and Callander 2002) show general conditions under which directed cycles of exchange provide stable configurations for optimal price formation as a market pricing mechanism.

[15] If an adjacency matrix has the upper rectangular property there are many ways to assign integer levels n_i to each node i such that for each line from i to j, $n_i > n_j$. The acyclic depth partition is only one algorithm to do so.

The DAG Structure in Ohta

To elaborate on the structural properties of the Ohta network, Figure 3 shows the outcome of a depth partition of the largest, weakly connected component of 4,500 firms. The partition reveals the simple hierarchical characteristics of this extremely complex web but rather merely ordering without any actual reduction of the data. There are seven linked but hierarchically ranked levels of firms. The vertical paths that connect the firms across the levels represent linked manufacturing processes that produce an enormous variety of parts, components, modules, and end products, with the result that the industry value-chain is similar to "food-chains" running within and beyond the Ohta ward.

Overall, the relatively low layers include large numbers of firms, and the relatively high layers have small numbers of firms. In these scalings, it appears that the depth hierarchy has multiple peaks of prominent OEMs at the top of the overlapping "mountains," or a series of "pyramids," which share numerous SME suppliers at lower layers.[16]

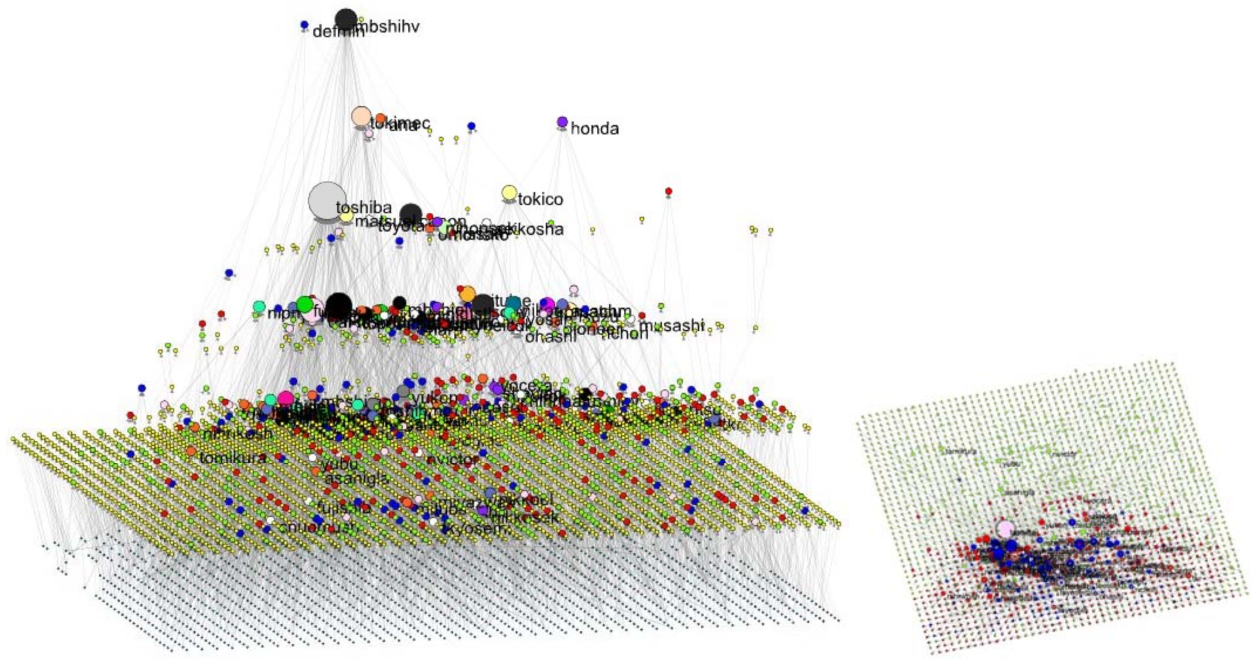

Fig. 3.—A global structure

Note.—The left graph shows subcontracting layers, according to the acyclic depth partition of the 4,500 firms in the main supplier/buyer component from the *Ohta-ku Akusesu Deta 1 & 2* (Ohta-ku Sangyo Shinko Kyokai 1997a; 1997b) data. The sizes of nodes reflect firm in-degree, or times listed by others as a prime buyer. The right drawing is a projection onto the x-y plane showing more of the network topology.

Instead of "neighborhood cohesion" measured by clustering in the small-world model, we argue that the "mountaintops" visible in the DAG structure of Figure 3 represent, as defined in Moody and White (2003), broader structural-embedding cohesion of the OEMs that organize the network from the top in order to achieve coordination and economic efficiency through the sharing of a thousands of dedicated SME suppliers down in the hierarchies. In the depth hierarchy, "neighborhood cohesion" in contrast would entail that, for

[16] The graphs assigned nodes randomly on the X-Y plane in a 3-D topology.

firms A, B, and C to form a completely connected triad, each would have to be located at a separate hierarchical level, by the mathematical dentition of the algorithm. In the industry value-chain, as organizing hubs fill a critical role typically as Tier-1 suppliers of OEMs at the top. Practically, two OEMs as prime buyers at the same level can possibly use their Tier-1 suppliers at the next lower level, as organizers of the subcontracting chains. There are other possible network configurations to represent cohesion in the depth hierarchies, as we discuss below. Thus, as to the organization of the DAG "mountaintops," the next hypothesis as follows:

Hypothesis 4. The "mountaintops" of the industrial district hierarchy are internally cohesive not as dense neighborhood clusters but by more distributed patterns of cohesion.

Analysis of Cohesion and Structural Embeddings

Why do modern production markets tend to have the structure of DAGs with upstream/downstream orientations in vertical hierarchies? Harrison White (2002a; 2002b) builds his theory of producers market with the observation that production chains are directional and hierarchically organized, and that mechanisms are in place for price profiles to emerge competitively. Our analysis of the acyclic depth partition of the largest component is consistent with the configurations, showing resemblance to depict the downstream sinks (inverted mountaintops) toward which goods and services flow in weavings through the complex, hierarchical network as entangled chains of processing activities.

Different measures of cohesion can help to distinguish different types of hierarchical structure from the viewpoint of underlying organizing principles of the network. As Powell, White, Koput and Owen-Smith (2005) show, forms of network analysis that incorporate cohesion analysis, as measurement of multiconnectivity of nodes, can help to identify powerful drivers or dynamical engines of a network, including those with hierarchical properties. White, Owen-Smith, Moody, and Powell (2005) identified a similar kind in a different large-scale network. While Granovetter (1985; 1992) calls this type of entangled weavings "economic embedding," White and Harary (2001) developed a similar but precise network concept and measure of structural cohesion for which Moody and White (2003) provided a computational algorithm that also computes a related measure of theirs, called *structural embedding*.

The large component of 4,500 Ohta firms, for example, contains a large bicomponent of 1,609 firms—structurally cohesive and robust as multiconnected at level 2. A *bicomponent* where all the nodes are connected by two or more independent paths is a measure of cohesion as the existence of the alternative path makes the bicomponent robust.[17] All of the top 105 prime buyers in terms of in-degree in the full network are contained in the bicomponent, and of the 97 firms in the bicomponent with in-degree of 5 or more, 88 are firms listed as the top 105 prime buyers for the whole network. A hierarchical nesting is apparent in the composition of the Ohta network.

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[17] A bicomponent (tricomponent, *k*-component, multiconnected component) is also defined as a maximal subgraph that cannot be disconnected by fewer than two (three, *k*) nodes. By Menger's theorem, this is equivalent to every pair of nodes having two (three, *k*) or more node-independent paths connecting them. This is also equivalent to a maximal subgraph having at least one cycle that includes every pair of nodes in the bicomponent when the directions of the arcs are disregarded.

Cohesion in Hierarchies and Assortative Correlation of Hubs

An acyclic depth hierarchy such as visualized in Figure 3—nodes assigned randomly on the XY plane in a 3D graph—does not necessarily entail that the top of this hierarchy is cohesively integrated. Taking into account the restrictions in data collection as up to three prime buyers, we need to evaluate, in probabilistic terms, whether there is a greater tendency toward cohesion in the distribution of node attachments as we move up the hierarchy.

We computed the ratios of node links to the number of nodes in the subsets of nodes, at each of the seven levels and above in the largest component of 4,500 firms, with the subsets as follows: Levels 1-7; 2-7; 3-7; 4-7; 5-7; and 6-7.[18] These ratios can be used to indicate the extent to which nodes as hubs are likely to form links with other hubs, as assortative corrlation. As Figure 4 shows, the first four bins for firms of levels 1 to 4, which constitute 94% of all nodes, fit a power-law decay (R^2 = 0.999), but those in levels 5-6 have more links to nodes at higher levels than expected by the decay curve. Thus, the firms at levels 5 and 6 have more connections with those at 6 and 7 than would be expected by power-law decay, which clearly is evidence of cohesive integration at the upper levels of the depth hierarchy. This graph clearly indicates *assortative correlation* among hubs of the network (Newman 2002).[19]

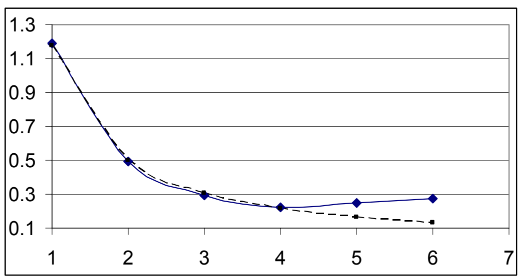


Fig. 4.—Assortative correlation for links in the upper depth layers of the largest component

Note.—A solid line shows actual number of links per node at each level; a dotted line shows this ratio as extrapolated from power-law decay.

Triads Census

A *triads* census, which was tested on both the largest component and the largest bicomponent, offers another way to measure the cohesive and hierarchical properties. Table 2 shows the nontrivial statistical results concerning occurrences of five critically relevant triad types, as follows: A prime buyer with two suppliers (021U); a supplier with two prime buyers (021D); vertical chains in the subcontracting tiers (021C); transitive vertical closures (030T); and directed cycles (030C).

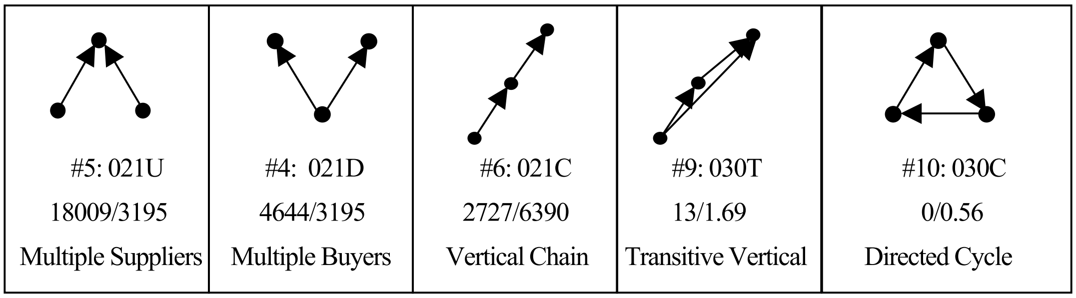
[18] The figure only shows six of the seven levels generated by the depth partition, as nodes at level 7 cannot have internal links because they are at the same level. Level 7 nodes are involved, however, in the assortative links originating at lower levels.

[19] Although we could compute the assortative correlation or "mixing" (Newman 2002) of whether links are more likely between hubs than would be expected from random oriented links holding constant the in- and out-degree distributions, we prefer this simpler measure because it will also display results that are not monotonic.

TABLE 2

Triad Census of Largest Component and Bicomponent in Comparison

	Largest Component (4500 nodes)			Largest Bicomponent (1609 nodes)		
Triad Types	Actual	Expected	Ratio (Actual / Expected)	Actual	Expected	Ratio (Actual / Expected)
5 021 U	18009	3195	5.637	11737	1882	6.236
4 021 D	4644	3195	1.454	1658	1882	0.881
6 021 C	2727	6390	0.428	1181	3766	0.314
9 030 T	13	1.68	7.692	13	3.59	3.621
10 030 C	0	0.56	0	0	1.19	0



First, the most frequent triad corresponds to that of multiple suppliers for a prime buyer, or 021U triad. As conceptualized by "flexible specialization" theory*,* the multiple suppliers triad happens in situations where either two suppliers are competing to be suppliers of a common prime buyer when they are engaged in the same area of specialized manufacturing processes, or alternatively, two dedicated suppliers are organized by a prime buyer to work as its suppliers in different specialized areas in the division of labor. These cases are often empirically observed that leading Japanese OEMs let a few suppliers engage in similar if not the same processing activities or parts and components manufacturing simultaneously in order to keep them compete in product quality and pricing as well as hedging and security purposes preparing for contingencies. There were 18,009 and 11,737 such triads in the Ohta component and bicomponent, respectively, more than 5.6 times many as expected by chance in the largest component ($p < 0.00000000000001$), and 6.2 times as many in the largest bicomponent ($p < 0.000000001$).[20]

Second, in contrast, the multiple buyers triad, labeled 021D in Table 2, happens when a supplier provides its processing services and goods to two prime buyers simultaneously that might be in direct competition in the same industry. There were 4,644 and 1,658 cases in the large component and bicomponent, respectively.[21] These numbers are only 1.45 and 0.88 times the expected frequency of such triads in the component and

[20] These probabilities may be estimated by a chi-squared test for $(\text{actual-expected})^2/\text{expected}$ in a triad sample size for a network with N nodes and $\binom{N}{3}$ triads.

[21] In most cases, our other analysis of the dataset indicated that this kind of "bridging" activity by the suppliers to work for two prime buyers, especially at the level of Tier-1 suppliers for prominent OEMs, was performed in limited cases. However, this bridging connections, possibly the Tier-1 taking advantage of "structural holes" in between the two competing OEMs, were more often seen in the light and heavy electric and consumer electronics manufacturing than in the automotive components production in Ohta although the actual numbers of such cases were relatively very small.

bicomponent respectively ($p < 0.01$ for the large component). This tendency is much weaker than the case for the multiple suppliers triad overall.

Compared to the general triads census, however, with an overall actual ratio of 4:1 in the multiple suppliers triads over the multiple buyers triads, we found a reversed trend 1:12 at the upper level of the hierarchy 5-7 by the depth partition. As seen in Figure 5, which depicts the node links at levels of 5–7, while the number of the firms at these high layers is very small, the reversal of ratios from 18,009:4,644 compared to 1:12 is statistically significant at $p < 0.000000000001$. This disparity, of course, occurs with the hubs of the network.[22]

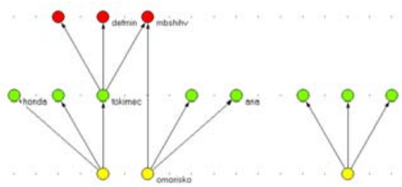


Fig. 5—Multiple prime buyers as dominant configuration at depth levels 5–7

Next, a path of two vertical links and no transitivity is very common. The vertical chain 021C occurs when a supplier indirectly supplies its products or processing services to a hierarchical distance-2 prime buyer through some value-adding role of the adjacent, intermediating prime buyer-supplier. The vertical chain is expected to occur by chance more often than the transitive vertical closure or 030T. Actually, the former occurred relatively less often than expected by chance in both the component and the bicomponent although the actual numbers of occurrence were relatively large, as there were 2,727 such occurrences for the component and 1,181 for the bicomponent.

By comparison, while the transitive vertical closure or 030T is expected to occur very rarely, it occurred by eight and four times more often than expected by chance in the component and the bicomponent, respectively. This difference is significant only at $p < 0.01$, only for the large component that involves only 13 transitive triples. Thus there is only a miniscule tendency for firms to form a transitive hierarchy, even though such a tendency is not avoided. This mild transitivity asserts itself to an insignificant degree that does not carry over to the bicomponent. The paucity of transitive triples contributes to the minimum level of clustering in the small-world model, as suppliers do not need the intermediating roles of their adjacent prime buyer-suppliers when they can work for the prime buyers directly, except in the case that the intermediary has very unique value-adding technologies.

Overall, the structural evidence for convergences toward a single hierarchy does exist, as the multiple suppliers triad is a much stronger tendency than the multiple prime buyers triad. But the reverse is true for the hubs of the network because the latter configuration tends to be concentrated in the upper levels of the hierarchy. With the triads census, the possible bias in the dataset introduced by listing only up to the three

[22] Although the actual number of firms at the levels of 5–7 as 48 is relatively small compared to the size of the component or 4,500, the finding from the triadic census are substantively very important, as these firms are the ones that are controlling the complex network, being positioned at the highest levels of the overall hierarchy.

prime buyers is controlled.[23]

CONCLUSIONS

The present research attempted to explain structural mechanisms of "flexible specialization" in a large-scale industrial district, applying some of the techniques of complex interactive network analysis. The initial hypothesis was that the complex web of regional supplier networks should fit the small-world formula where nodes are connected as a large sparse network (LSN) with its local clustering and short average path length. The small-world model was rejected on three of the four criteria, as follows: The network has a hierarchical structure in the scale-free degree distributions of hubs; the average shortest distance was too efficient to be generated by random rewirings; and clustering was mostly absent, and was considerably less than expected by chance. Thus, the underlying structural mechanisms that created the giant component of 4,500 firms would necessarily be very different from those generating the typical small-world phenomenon. While these initial results pointed at minimum to some hub-based integration mechanisms, the scale-free model also had to be rejected because the "bottom-up" mechanisms it posited were incorrect, as the complex system is primarily organized by hubs and leading prime buyers, as the "top-down" control is a nature of the subcontracting supplier-prime buyer relationships.

As an alternative explanation, the acyclic depth partition of the large component in the network of oriented supplier-prime buyer ties unveiled integrating mechanisms present in an industrial district from a different approach, not as local clustering based on linkages among Tier-1 suppliers as suggested by the small-world formula, but a broader level of cohesion within the hierarchically structured supplier networks of lead OEMs. Moreover, at the global level, the interfirm dynamics that seem to be operative in the entangled chains of linked manufacturing processes—a kind of industry value-chain composed of thousands of specialized SME suppliers and their prime buyers—are ones that generate a giant component, linking the OEMs' supplier group networks. From the viewpoint of enmeshed flows of goods and services, these mechanisms in combination formed a "multiple but inter-cohesive mountaintops" style structure where the overlapping sub-networks organized under prominent OEMs shared thousands of SME suppliers down in the subcontracting hierarchy.

Yet, deeper in the complex hierarchies, there was a structurally cohesive and highly integrated core of the network, comprised of simple bicomponent cohesion that acted as a powerful organizing principle of the node links. This single hierarchy thesis was also largely supported by probing the largest component and bicomponent, especially with the analyses of the different measures of cohesion that detected their structural properties such as the assortative correlation among the hubs and the discrete patterns of triadic network configurations in the interfirm trading. Specifically, the triads census found that there was a strong overall tendency for multiple suppliers to work for a prime buyer, relative to the minor tendency for a supplier to work

[23] Note that the probabilistic model for testing statistical significance of triads from the null hypothesis differs from that used for local clustering. The two tests disagree as to whether the fully connected triads are more or less likely than expected by chance. The ratio of fully connected triads to connected triples (13/25,390) from Table 2, however, does agree closely with the clustering coefficient 0.0000517 from Table 1. Newman, Strogatz, and Watts (2001) provide probability models for networks with arbitrary degree distributions.

for multiple prime-buyers. While the ratio was four to one in the largest component, this tendency reversed itself among the top OEMs or hubs in the hierarchy.

A major contribution of this present study is our finding that complex networks of production chains embedded in large-scale industrial districts should be acyclically hierarchical. It marks a first attempt to capture the interfirm dynamics deeply embedded in the complex webs. We provided statistical evidence that, as the number of prime buyers that each of these suppliers has appears to be quite small on the average in the LSNs in general, these production chain network structures of "flexible specialization," which merged for the "mass customization" after the 1990s, should form a directed acyclic graph, or DAG, with their limited number of overall depth layers,[24] and probably also with a cohesive core among extremely powerful OEMs and their "elite" suppliers as a generative, integrating, and organizing driver of the network. Thus, the DAG or depth hierarchy is likely to be the general property of the LSNs in the complex regional production systems, probably in later stages of their evolution, as an alternative model in lieu of the small-world and the scale-free network models.

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APPENDIX A: On the Actual Number of Buyers in Ohta—Some Evidence from Existing Fieldwork Studies

While it is difficult to assess the actual numbers of suppliers and buyers that each firm in Ohta actually possessed at the time of 1994-95 survey, it should be firm-size dependent to a certain degree. Whittaker (1997:78) reports that for Ohta, in 1990, a quarter of the micro factories had direct subcontracting relationships to factories with 100 employees or more, which are likely to include their prime buyers. According to data from another survey in 1997 (Chushokigyo Kenkyu Center 1999), among 1,341 SMEs in Ohta, given the response rate at 19.6% of the 6,853 firms, the SME suppliers with three employees or less typically had two to three buyers, while those with 30 or more usually engaged 100 buyers in their repeated business transactions. As a caveat, these buyers are not necessarily their prime buyers. On the flip side, SMEs with three employees or less had two to three suppliers of their own, while those with 30 or more kept 10 to 19 suppliers. Overall, approximately 70% of the SMEs in Ohta had two to three firms to receive and to place orders with one another.

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[24] Our statistical evidence and its empirical finding of the limited number of depth layers in the LSN is consistent with the recent simulation results by Weisbush and Battison (2005) although their lattice-type, conceptual model of production market differs substantively from our network integration model, which converges towards the cohesive core.

The ties identified from the present 1994-95 dataset possess the following structural characteristics. First, while the numbers from the 1997 data aforementioned are consistent with the 1994-95 data, the limit of up to three prime buyers, imposed on all 5,111 respondents, could be much less than the numbers of all prime buyers that otherwise could have been listed, especially by relatively large suppliers in Ohta. As the percentage of firms with 30 or more employees in 1990 was less than 4.6% of all the establishments in the industrial district, however, and approximately 80% of the firms in Ohta had nine or fewer employees, the impact of the restriction of up to three prime buyers should not be that great on the average number of all buyers. Had free listing been allowed, this average might be as high as 6-8 per firm as opposed to our estimate of 1.62-1.89 prime buyers. As a reminder, the term "all buyers" should generally be much more inclusive or looser than the "prime buyers" used for our dataset. Had numbers of all buyers been collected and still no directed cycles occurred in these data, this would be a highly significant difference from what would be expected by chance.

APPENDIX B: Estimating the Expected Number of Directed Cycles

When does (almost) every large sparse oriented (LSO) network become a directed acyclic graph (DAG)? An LSO network is one with antisymmetric links that are sparse in relation to the large number of nodes. Lacking directed cycles, the network becomes a DAG, or directed acyclic graph, that will allow an acyclic depth partition into levels. The minimum number of levels such that all directed links go from a lower to a higher level is the length of the longest directed path in the DAG. Having identified a structural property of the supplier-prime buyer networks in Ohta as DAG, our next question was: What are the conditions of the minimal sparsity and the size in terms of numbers of nodes that almost every LSO net satisfies? In other words, under what conditions is a DAG a close approximation to the structure of an LSO network?

Imagine for example an LSO net of 2,000 nodes and 4,000 arcs in which there is a single directed cycle of length three, when 4.78 directed cycles are expected by random wiring of every link. The network is not a DAG, but as a model of network structure, eliminating an incoming edge to one node, the network becomes a DAG with one observed error.

The actual number of prime buyers that each of the suppliers in Ohta possessed is generally very small, as described in Appendix A. We considered the number and average total degree of the nodes in a generative probabilistic model: Calculate the total number of sets (*k*-tuples) of nodes of length 3 (triads), 4, up to *n*; for each *k*-tuple size, and to get the expected frequency of a cycle, multiply these frequencies by the probability of an undirected cycle at each length times the probability that the alignments of edge direction make a directed cycle ($1/2^{(k-2)}$). For an oriented triple, for example, the chance that the arcs for a directed cycle is ½. Summed over the types of *k*-tuples, we get the expected frequencies of cycles at all possible lengths, as Figure B1 shows.

Let n be the number of nodes in an oriented (directed) network and N be the number of directed (antisymmetric) edges. For LSO networks, n is large, and the average degree of nodes, $deg = N/n$, is small. The following computations, written by Aleš Ziberna (University of Ljubljana) at our request, are in R code for computing the expected frequency of a directed cycle of length k=3 or more for a network with $deg = N/n$. (To test these equations, open R, paste the function, and press return).

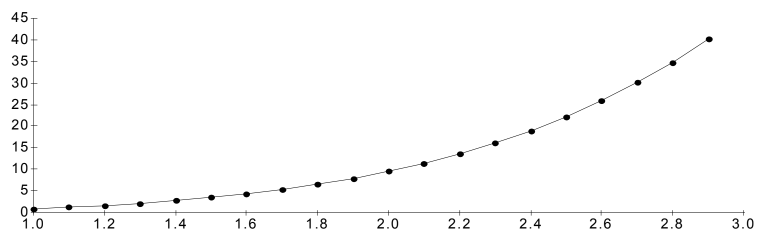
```
cyc<-function(n,deg,k){prob<-function(n,deg,k)
```

```
        {       choose(n,k)*((2*deg/(n-1))^k) /(2^(k-2))        }
        sum(prob(n,deg,k=k))                                    }
cyc(n=1609,deg=1.6,k=3:30)
                                                                        result=4.2889
cyc<-function(n,deg,k){prob<-function(n,deg,k)
        {       choose(n,k)*((2*deg/(n-1))^k) /(2^(k-2))        }
        sum(prob(n,deg,k=k))                                    }
cyc(n=9609,deg=1.6,k=3:30)
                                                                        result=4.2919
```

Results of these computations are very close (differing at the 4th digit, i.e., only by 0.07 of one percent) for all but very small networks. Iterating this computation for different size *n* and average *deg*, a distribution of the expected number of directed cycles by the average out-degree level approximates a power law ($R^2 = 0.9991$) for large networks that is independent of sample size:

$$y = 0.815\, x^{3.6}$$

Where y is the expected number of directed cycles and x is average out-degree. The prediction is that when average degree is less than 2, the number of expected cycles is very low. For the Ohta bicomponent of 1,609 firms, the average degree is 1.53, and the expected number of cycles is 3.67.



Average Out-Degree	1.0	1.1	1.2	1.3	1.4	1.5	1.6	1.7	1.8	1.9
Expected # of Directed Cycles	0.872	1.196	1.6	2.096	2.698	3.422	4.288	5.308	6.51	7.912
Average Out-Degree	2.0	2.1	2.2	2.3	2.4	2.5	2.6	2.7	2.8	2.9
Expected # of Directed Cycles	9.54	11.422	13.594	16.084	18.93	22.178	25.87	30.058	34.7982	40.154

Fig. B1.— Expected number of directed cycles in a large sparse oriented graph by average out-degree

APPENDIX C: Estimating the Out-Degree Distribution for Prime Buyers

The densest portion of the industrial district network is the bicomponent consisting of 1,609 firms. The out-degree distribution for the bicomponent showed 483 firms with no buyers, 127 with one, 663 with two, and 336 with three or more. The modal number is two for the entire distribution. Assuming that the frequency distribution for firms with two or more prime buyers is a declining function that follows either an exponential or power-law decline from 663 to a missing tail totaling 336 firms with three or more prime buyers, Figure C1 shows the two fitted distributions that match these requirements, calculated for the network size of the largest bicomponent. The two curves match declining tails for 663 firms with two prime buyers and a total of 336 firms with three or more prime buyers, first using an exponentially declining tail, and second using power-law decline in the tail of the distribution. The average out-degree for the fitted exponential distribution is 1.62 and that for the power-law distribution is 1.89. In the first case, that of the exponential distribution, the expected number of directed cycles is 2.2. In the second, for the power-law distribution case, the expected number is 3.8. Thus, the expected number of directed cycles, given that there is avoidance of clustering, as found in testing the small-world graph, is very low in each case.

Note that these estimates apply to the question of "prime buyers" and not to that of "please list all of your buyers," in which case the numbers and averages would be considerably greater, as discussed in Appendix A. We also tried to put bounds on estimates for numbers and averages of total buyers from the evidence of other surveys, and we come up with averages between 6 and 8 buyers. But the surveys we use indicate that smaller firms have very few buyers, and there are very few larger firms with large numbers of buyers.

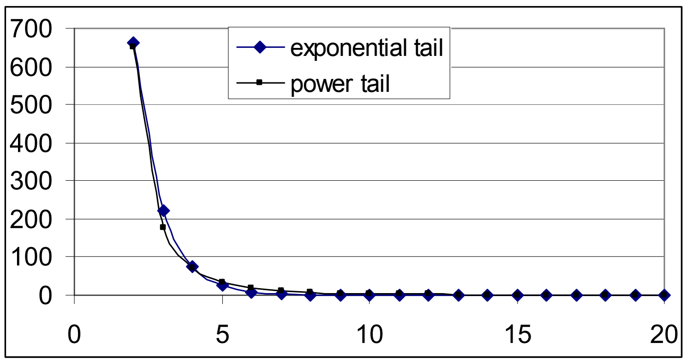


Fig. C1.— Number of firms by out-degree levels, with fitted exponential and power-law decay functions

We draw two conclusions from this finding. First, even if we had complete data on *all* the buyers for the supplier-buyer relationships, there would be 0-2 directed cycles, most likely of length three, even if the data were random. Second, under this null hypothesis, there is no reason to expect that full data would produce so many directed cycles that the acyclic depth hierarchy would not be an appropriate model of network structure. Had we found 1-2 directed cycles of length 3, these 1-2 sets of three firms would simply be placed in the same level of the hierarchy.

As a recap, when might almost every large-scale directed network have a DAG if only because of a high degree of randomness in the distribution of oriented links? The simple answer is that this will occur when the average degree of nodes is under 2 or, more precisely, under some limit that is less than 2. Because the Ohta supplier network has an average degree of well below 2, we would expect Hypotheses 2 to be supported—existence of a depth hierarchy—even if it were a simple random oriented graph with the same average degree. To be precise, for an oriented network with an average degree of 1.19 per node, as in the largest component of the Ohta graph, the expected number of directed cycles is very low, less than 1 expected cycle, as shown earlier. Large networks from other industrial districts, if their supplier-buyer graphs were oriented, could easily have similar expectations of being acyclic graphs if their average degree were low. Thus, Hypothesis 3—DAG as the general property of the supplier-prime buyer networks in large-scale industrial districts—is supported, as the general applicability of DAG structure is statistically proved.

If the network were clustered rather than random, as in the small-world hypothesis, we would expect many cycles and thus many directed cycles in the network. In this case, or if there were clustering, as in the small-world model, absence of cycles would be statistically significant. In fact, as seen above in Table 2, the clustering coefficient is 10 times *smaller* than expected in a random graph. This makes it even less likely that there will be directed cycles of length 3 in the large sparse directed network of the industrial district. Thus, the *absence* of the small-world property with respect to clustering is relevant to the likelihood that an oriented graph will also be acyclic and thus hierarchical in structure.

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